Radiative natural supersymmetry with mixed axion/higgsino cold dark matter

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Models of natural supersymmetry seek to solve the little hierarchy problem by positing a spectrum of light higgsinos $\stackrel{<}{\sim}$ 200 GeV and light top squarks $\stackrel{<}{\sim}$ 500 GeV along with very heavy squarks and TeV-scale gluinos. Such models have low electroweak finetuning and are safe from LHC searches. However, in the context of the MSSM, they predict too low a value of m_h and the relic density of thermally produced higgsino-like WIMPs falls well below dark matter (DM) measurements. Allowing for high scale soft SUSY breaking Higgs mass $m_{H_u} > m_0$ leads to natural *cancellations* during RG running, and to radiatively induced *low finetuning at the electroweak scale*. This model of *radiative natural SUSY* (RNS), with large mixing in the top squark sector, allows for finetuning at the 5-10% level with TeV-scale top squarks and a 125 GeV light Higgs scalar h. If the strong CP problem is solved via the PQ mechanism, then we expect an axion-higgsino admixture of dark matter, where either or both the DM particles might be directly detected.

I. INTRODUCTION

The recent fabulous discovery by Atlas and CMS of a Higgs-like resonance at 125 GeV[1, 2] adds credence to supersymmetric models (SUSY) of particle physics in that the mass value falls squarely within the narrow predicted MSSM window: $m_h \sim 115 - 135$ GeV[3]. At the same time, a lack of a SUSY signal at LHC7 and LHC8 implies squarks and gluinos beyond the 1 TeV range[4, 5], exacerbating the *little hierarchy problem* (LHP):

- how do multi-TeV values of SUSY model parameters conspire to yield a Z-boson mass of just 91.2 GeV?

Models of *natural supersymmetry*[6] address the LHP by positing a spectrum of light higgsinos $\stackrel{<}{\sim}$ 200 GeV and light top squarks $\stackrel{<}{\sim}$ 500 GeV along with very heavy first/second generation squarks and TeV-scale gluinos[7–9]. Such a spectrum allows for low electroweak finetuning while at the same time keeping sparticles safely beyond LHC search limits. In these models, the radiative corrections to m_h, which increase with $m_{\tilde{t}_i}^2$, are somewhat suppressed and have great difficulty in generating a light SUSY Higgs scalar with mass $m_h \sim 125$ GeV[10]. Thus, we are faced with a new conundrum: how do we reconcile low electroweak finetuning with such a large value of m_h[11]? In addition, the light higgsino-like WIMP particles predicted by models of natural SUSY lead to a thermally-generated relic density which is typically a factor 10-15 below[9, 12] the WMAP measured value of $\Omega_{CDM}h^2 \simeq 0.11$.

One solution to the finetuning/Higgs problem is to add extra matter to the theory, thus moving beyond the MSSM[11]. For example, adding an extra singlet as in the NMSSM adds further quartic terms to the higgs potential thus allowing for increased values of m_h[13]. One may also add extra vector-like matter to increase m_h while maintaining light top squarks[14]. In the former case of the NMSSM, adding extra gauge singlets may lead to re-introduction of destabilizing divergences into the theory[15]. In the latter case, one might wonder about the ad-hoc introduction of extra weak scale matter multiplets and how they might have avoided detection A third possibility, which is presented below, is to re-examine EWFT and to ascertain if there do exist sparticle spectra *within the MSSM* that lead to $m_h \sim 125$ GeV while maintaining modest levels of electroweak finetuning.

II. ELECTROWEAK FINETUNING

One way to evaluate EWFT in SUSY models is to examine the minimization condition on the Higgs sector scalar potential which determines the Z boson mass. (Equivalently, one may examine the mass formula for m_h and draw similar conclusions.) One obtains the well-known tree-level expression

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 . \tag{1}$$

To obtain a *natural* value of M_Z on the left-hand-side, one would like each term C_i (with $i = H_d$, H_u and μ) on the right-hand-side to be of order $m_Z^2/2$. This leads to a finetuning parameter definition

$$\Delta \equiv max_i \left(C_i \right) / (m_Z^2/2) \tag{2}$$

where $C_{H_u} = |-m_{H_u}^2 \tan^2 \beta/(\tan^2 \beta - 1)|/$, $C_{H_d} = |m_{H_d}^2/(\tan^2 \beta - 1)|/$ and $C_\mu = |-\mu^2|$. Since C_{H_d} is suppressed by $\tan^2 \beta - 1$, for even moderate $\tan \beta$ values this expression reduces approximately to

$$\frac{m_Z^2}{2} \simeq -m_{H_u}^2 - \mu^2. \tag{3}$$

The question then arises: what is the model, what are the input parameters, and how do we interpret Eq's 1 and 3?

Suppose we have a model with input parameters defined at some high scale $\Lambda \gg m_{SUSY}$, where m_{SUSY} is the SUSY breaking scale ~ 1 TeV. Then

$$m_{H_u}^2(m_{SUSY}) = m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 \qquad (4)$$

where

$$\delta m_{H_u}^2 \simeq -\frac{3f_t^2}{8\pi^2}\left(m_{Q_3}^2 + m_{U_3}^2 + A_t^2\right)\ln\left(\frac{\Lambda}{m_{SUSY}}\right). \quad (5)$$

The usual lore is that in a model defined at energy scale Λ, then both $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$ must be of order $m_Z^2/2$ in order to avoid finetuning. In fact, requiring $\delta m_{H_u}^2 \stackrel{<}{\sim} m_Z^2/2$ has been used to argue for a sparticle mass spectra of natural SUSY. Taking $\Delta = 10$ corresponds to

- $|\mu| \stackrel{<}{\sim} 200$ GeV,

- $m_{\tilde{t}_i},\ m_{\tilde{b}_1} \stackrel{<}{\sim} 500$ GeV,

- $m_{\tilde{g}} \stackrel{<}{\sim} 1.5$ TeV.

Since first/second generation squarks and sleptons hardly enter into Eq. 1, these can be much heavier: beyond LHC reach and also possibly providing a (partial) decoupling solution to the SUSY flavor and CP problems:

- $m_{\tilde{q},\tilde{\ell}} \sim 10 - 50$ TeV.

The natural SUSY solution reconciles lack of a SUSY signal at LHC with allowing for electroweak naturalness. It also predicts that the $\tilde{t}_{1,2}$ and $\tilde{b}_1$ may soon be accessible to LHC searches. New limits from direct top and bottom squark pair production searches, interpreted within the context of simplified models, are biting into the NS parameter space[16]. Of course, if $m_{\tilde{t}_{1,2}},\ m_{\tilde{b}_1} \simeq m_{\widetilde{Z}_1}$, then the visible decay products from stop and sbottom production will be soft and difficult to see at LHC. A more worrisome problem is that, with such light top squarks, the radiative corrections to m_h are not large enough to yield $m_h \simeq 125$ GeV. This problem has been used to argue that additional multiplets beyond those of the MSSM must be present in order to raise up m_h while maintaining very light third generation squarks[11]. A third issue is that the relic abundance of higgsino-like WIMPs, calculated in the standard MSSM-only cosmology, is typically a factor 10-15 below measured values. These issues have led some people to grow increasingly skeptical of weak scale SUSY, even as occurs in the natural SUSY incarnation.

One resolution to the above finetuning problem is to merely invoke a SUSY particle spectrum at the weak scale, as in the pMSSM model. Here, $\Lambda \sim m_{SUSY}$ so $\delta m_{H_u}^2 \sim 0$ and we may select parameters $m_{H_u}^2 \sim \mu^2 \sim m_Z^2$. While a logical possibility, this solution avoids the many attractive features of a model which is valid up to

a high scale such as $\Lambda \sim m_{GUT}$, with gauge coupling unification and radiative electroweak symmetry breaking driven by a large top quark mass.

Another resolution is to impose Eq. 1 as a condition on high scale models, but using only weak scale parameters. In this case, we will differentiate the finetuning measure as Δ_{EW}, while the finetuning measure calculated using high scale input parameters we will refer to as Δ_{HS}. The weaker condition of allowing only for $\Delta_{EW} \stackrel{<}{\sim} 20$ allows for possible *cancellations* in $m_{H_u}^2(m_{SUSY})$. This is precisely what happens in what is known as the hyperbolic branch or focus point region of mSUGRA[17]: $m_{H_u}^2(\Lambda) \simeq -\delta m_{H_u}^2 \sim m_Z^2$ and consequently a value of $\mu^2 \sim m_Z^2$ is chosen to enforce the measured value of m_Z from Eq. 1.[1] The HB/FP region of mSUGRA occurs for small values of trilinear soft parameter A_0. Small A_0 leads to small A_t at the weak scale, which leads to $m_h \sim 115 - 120$ GeV, well below the Atlas/CMS measured value of $m_h \simeq 125$ GeV. Scans over parameter space show the HB/FP region is nearly excluded if one requires both low $|\mu| \sim m_Z$ and $m_h \sim 123 - 127$ GeV[19, 20].

The cancellation mechanism can also be seen from an approximate analytic solution of the EWSB minimation by Kane *et al.*[21]:

$$m_Z^2 \simeq\ = -1.8\mu^2 + 5.9M_3^2 - 0.4M_2^2 - 1.2m_{H_u}^2 + 0.9m_{Q_3}^2$$
$$+0.7m_{U_3}^2 - 0.6A_tM_3 + 0.4M_2M_3 + \cdots \qquad (6)$$

(which adopts $\tan\beta = 5$ although similar expressions may be gained for other $\tan\beta$ values). All parameters on the RHS are GUT scale parameters. We see one solution for obtaining m_Z on the left-hand side is to have all GUT scale parameters of order m_Z (this is now excluded by recent LHC limits). The other possibility– if some terms are large (like $M_3 \stackrel{>}{\sim} 0.4$ TeV in accord with recent LHC limits)– is to have large cancellations. The simplest possibility– using $M_3 \stackrel{>}{\sim} 0.4$ TeV– is then to raise up $m_{H_u}^2$ beyond m_0 such that there is a large cancellation. This possibility is allowed in the non-universal Higgs models[22, 23].

In mSUGRA, the condition that $m_{H_u}^2 = m_{H_d}^2 = m_{\tilde{q}}^2 = m_{\tilde{\ell}}^2 \equiv m_0^2$ at the high scale is anyways hard to accept. One might expect that all matter scalars in each generation are nearly degenerate since the known matter in each generation fills out complete 16-dimensional representations of $SO(10)$. However, the distinguishing feature of the Higgs multiplets is that they would live in 10-dimensional representations, and we then would *not* expect $m_{10} = m_{16}$ at m_{GUT}. A more likely choice would be to move to the non-universal Higgs model, which comes in several varieties. It was shown in Ref. [23] that by

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[1] This may also occur in other varied models such as mixed moduli-AMSB[18].

adopting a simple one-parameter extension of mSUGRA– known as the one extra parameter non-universal Higgs model (NUHM1), with parameter space

$$m_{H_u}^2 = m_{H_d}^2 \equiv m_\phi^2, \; m_0, \; m_{1/2}, \; A_0, \; \tan\beta, \; sign(\mu) \quad (7)$$

– then for any spectra one may raise m_ϕ up beyond m_0 until at some point $m_{H_u}^2(m_{SUSY})$ drops in magnitude to $\sim m_Z^2$. The EWSB minimization condition then also forces $|\mu| \sim m_Z$. The worst of the EWFT is eliminated due to a large cancellation between $m_{H_u}^2(\Lambda = m_{GUT})$ and $\delta m_{H_u}^2$ leading to low Δ_{EW} and a model which enjoys electroweak naturalness.

The cancellation obviously can also be implemented in the 2-extra-parameter model NUHM2 where both $m_{H_u}^2(m_{GUT})$ and $m_{H_d}^2(m_{GUT})$ may be taken as free parameters (as in an $SU(5)$ SUSY GUT) or– using the EWSB minimization conditions– these may be traded for weak scale values of μ and m_A as alternative inputs[22]. A third possibility that will allow for an improved decoupling solution to the SUSY flavor and CP problems would occur if we allow for split generations $m_0(3)$ and $m_0(1) \simeq m_0(2) \equiv m_0(1,2)$ (SGNUHM). The latter condition need not require exact degeneracy, since with $m_0(1) \sim m_0(2) \sim 10 - 20$ TeV we obtain only a *partial* decoupling solution to the flavor problem. Taking $m_0(1,2) \sim 10-20$ TeV solves the SUSY CP problem[24].

III.   RADIATIVE NATURAL SUSY

Motivated by the possibility of cancellations occuring in $m_{H_u}^2(m_{SUSY})$, we go back to the EWSB minimization condition and augment it with radiative corrections Σ_u^u and Σ_d^d since if $m_{H_u}^2$ and μ^2 are suppressed, then these may dominate:

$$\frac{m_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \; . \quad (8)$$

Here the Σ_u^u and Σ_d^d terms arise from derivatives of the *radiatively corrected* Higgs potential evaluated at the potential minimum. At the one-loop level, Σ_u^u contains the contributions[25] $\Sigma_u^u(\tilde{t}_{1,2})$, $\Sigma_u^u(\tilde{b}_{1,2})$, $\Sigma_u^u(\tilde{\tau}_{1,2})$, $\Sigma_u^u(\widetilde{W}_{1,2})$, $\Sigma_u^u(\widetilde{Z}_{1-4})$, $\Sigma_u^u(h, H)$, $\Sigma_u^u(H^\pm)$, $\Sigma_u^u(W^\pm)$, $\Sigma_u^u(Z)$, and $\Sigma_u^u(t)$. Σ_d^d contains similar terms along with $\Sigma_d^d(b)$ and $\Sigma_d^d(\tau)$ while $\Sigma_d^d(t) = 0$ [26]. There are also contributions from D-term contributions to first/second generation squarks and sleptons which nearly cancel amongst themselves (due to sum of weak isospins/hypercharges equaling zero). Once we are in parameter space where $m_{H_u}^2(m_{SUSY}) \sim \mu^2 \sim m_Z^2$, then the radiative corrections Σ_u^u may give the largest contribution to Δ_{EW}.

The largest of the Σ_u^u almost always come from top

squarks, where we find

$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \times$$
$$\left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8g_Z^2(\frac{1}{4} - \frac{2}{3}x_W)\Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right] (9)$$

where $\Delta_t = (m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)/2 + m_Z^2 \cos 2\beta(\frac{1}{4} - \frac{2}{3}x_W)$, $g_Z^2 = (g^2 + g'^2)/8$, $x_W \equiv \sin^2\theta_W$ and $F(m^2) = m^2 \left(\log(m^2/Q^2) - 1\right)$. In Ref. [27], it is shown that for the case of the $\tilde{t}_1$ contribution, as $|A_t|$ gets large there is a suppression of $\Sigma_u^u(\tilde{t}_1)$ due to a cancellation between terms in the square brackets of Eq. (9). For the $\tilde{t}_2$ contribution, a large splitting between $m_{\tilde{t}_2}$ and $m_{\tilde{t}_1}$ yields a large cancellation within $F(m_{\tilde{t}_2}^2)$ $\left(\log(m_{\tilde{t}_2}^2/Q^2) \to \log(m_{\tilde{t}_2}/m_{\tilde{t}_1}) \to 1\right)$ for $Q^2 = m_{\tilde{t}_1}m_{\tilde{t}_2}$, leading also to suppression. So while large $|A_t|$ values suppress both top squark contributions to Σ_u^u, at the same time they also lift up the value of m_h, which is near maximal for large, negative A_t. Combining all effects, one sees that the same mechanism responsible for boosting the value of m_h into accord with LHC measurements can also suppress the Σ_u^u contributions to EWFT, leading to a model with electroweak naturalness.

To illustrate these ideas, we adopt a simple benchmark point from the 2-parameter non-universal Higgs mass SUSY model NUHM2[22], but with split generations, where $m_0(3) < m_0(1,2)$. In Fig. 1, we take $m_0(3) = 5$ TeV, $m_0(1,2) = 10$ TeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$ with $\mu = 150$ GeV, $m_A = 1000$ GeV and $m_t = 173.2$ GeV. We allow the GUT scale parameter A_0 to vary, and calculate the sparticle mass spectrum using Isajet 7.83[28], which includes the new EWFT measure. In frame $a)$, we plot the value of m_h versus A_0. While for $A_0 \sim 0$ the value of $m_h \sim 120$ GeV, as A_0 moves towards $-2m_0(3)$, the top squark radiative contributions to m_h increase, pushing its value up to 125 GeV. (There is an expected theory error of ± 2 GeV in our RGE-improved effective potential calculation of m_h, which includes leading 2-loop effects[29].) At the same time, in frame $b)$, we see the values of $m_{\tilde{t}_{1,2}}$ versus A_0. In this case, large values of A_0 suppress the soft terms $m_{Q_3}^2$ and $m_{U_3}^2$ via RGE running. But also large weak scale values of A_t provide large mixing in the top squark mass matrix which suppresses $m_{\tilde{t}_1}$ and leads to an increased splitting between the two mass eigenstates which suppresses the top squark radiative corrections $\Sigma_u^u(\tilde{t}_2)$. The EWFT measure $\Delta \equiv \Delta_{EW}$ is shown in frame $c)$, where we see that while $\Delta \sim 50$ for $A_0 = 0$, when A_0 becomes large, then Δ drops to 10, or $\Delta^{-1} = 10\%$ EWFT. In frame $d)$, we show the weak scale value of A_t versus A_0 variation. While the EWFT is quite low– in the range expected for natural SUSY models– we note that the top squark masses remain above the TeV level, and in particular $m_{\tilde{t}_2} \sim 3.5$ TeV, in contrast to previous natural SUSY expectations.
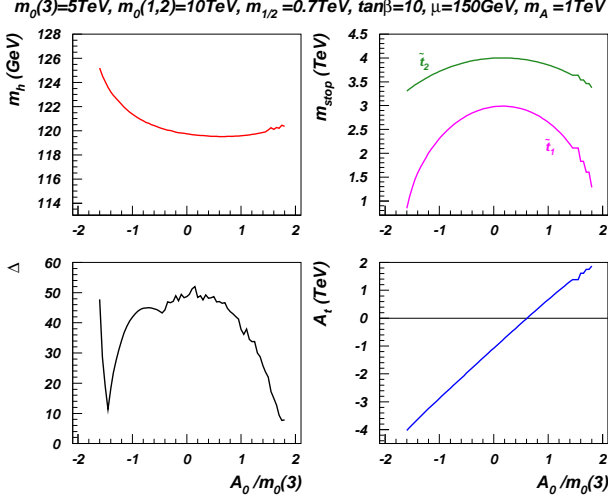
FIG. 1: Plot of *a)* m_h, *b)* $m_{\tilde{t}_{1,2}}$, *c)* Δ and *d)* A_t versus variation in A_0 for a model with $m_0(1,2) = 10$ TeV, $m_0(3) = 5$ TeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$ and $\mu = 150$ GeV and $m_A = 1$ TeV.

IV. SPARTICLE SPECTRUM

The sparticle mass spectrum for this radiative NS benchmark point (RNS1) is shown in Table I for $A_0 = -7300$ GeV. The heavier spectrum of top and bottom squarks seem likely outside of any near-term LHC reach, although in this case gluino[30] and possibly heavy electroweak-ino[31] pair production may be accessible to LHC14. Dialing the A_0 parameter up to -8 TeV allows for $m_h = 125.2$ GeV but increases EWFT to $\Delta = 29.5$, or 3.4% fine-tuning. Alternatively, pushing m_t up to 174.4 GeV increases m_h to 124.5 GeV with 6.2% fine-tuning; increasing $\tan\beta$ to 20 increases m_h to 124.6 GeV with 3.3% fine-tuning. We show a second point RNS2 with $m_0(1,2) = m_0(3) = 7.0$ TeV and $\Delta = 11.5$ with $m_h = 125$ GeV; note the common sfermion mass parameter at the high scale. For comparison, we also show in Table I the NS2 benchmark from Ref. [9]; in this case, a more conventional light spectra of top squarks is generated leading to $m_h = 121.1$ GeV, but the model– with $\Delta = 23.7$– has higher EWFT than RNS1 or RNS2.

The RNS model shares some features of generic NS models, but also includes important differences. The several benchmark points shown in Table I imply that RNS is characterized by:

- a higgsino mass $\mu \sim M_Z \sim 100 - 300$ GeV,

- a light top squark $m_{\tilde{t}_1} \sim 1 - 2$ TeV,

- a heavier top-squark $m_{\tilde{t}_2} \sim e\, m_{\tilde{t}_1} \sim 2 - 4$ TeV (here, $e \equiv 2.718...$),

- $m_{\tilde{g}} \sim 1 - 5$ TeV,

- first/second generation sfermions $\sim 5 - 20$ TeV.

parameter	RNS1	RNS2	NS2
$m_0(1,2)$	10000	7025.0	19542.2
$m_0(3)$	5000	7025.0	2430.6
$m_{1/2}$	700	568.3	1549.3
A_0	-7300	-11426.6	873.2
$\tan\beta$	10	8.55	22.1
μ	150	150	150
m_A	1000	1000	1652.7
$m_{\tilde{g}}$	1859.0	1562.8	3696.8
$m_{\tilde{u}_L}$	10050.9	7020.9	19736.2
$m_{\tilde{u}_R}$	10141.6	7256.2	19762.6
$m_{\tilde{e}_R}$	9909.9	6755.4	19537.2
$m_{\tilde{t}_1}$	1415.9	1843.4	572.0
$m_{\tilde{t}_2}$	3424.8	4921.4	715.4
$m_{\tilde{b}_1}$	3450.1	4962.6	497.3
$m_{\tilde{b}_2}$	4823.6	6914.9	1723.8
$m_{\tilde{\tau}_1}$	4737.5	6679.4	2084.7
$m_{\tilde{\tau}_2}$	5020.7	7116.9	2189.1
$m_{\tilde{\nu}_\tau}$	5000.1	7128.3	2061.8
$m_{\widetilde{W}_2}$	621.3	513.9	1341.2
$m_{\widetilde{W}_1}$	154.2	152.7	156.1
$m_{\widetilde{Z}_4}$	631.2	525.2	1340.4
$m_{\widetilde{Z}_3}$	323.3	268.8	698.8
$m_{\widetilde{Z}_2}$	158.5	159.2	156.2
$m_{\widetilde{Z}_1}$	140.0	135.4	149.2
m_h	123.7	125.0	121.1
$\Omega^{std}_{\widetilde{Z}_1}h^2$	0.009	0.01	0.006
$BF(b \to s\gamma) \times 10^4$	3.3	3.3	3.6
$BF(B_s \to \mu^+\mu^-) \times 10^9$	3.8	3.8	4.0
$\sigma^{SI}(\widetilde{Z}_1 p)$ (pb)	1.1×10^{-8}	1.7×10^{-8}	1.8×10^{-9}
Δ	9.7	11.5	23.7

TABLE I: Input parameters and masses in GeV units for two *radiative natural SUSY* benchmark points and one NS point with $\mu = 150$ GeV and $m_t = 173.2$ GeV.

While $\mu \sim m_Z$ as in usual NS models, the heavier top squarks and gluinos implied by RNS allow for $m_h \simeq 125$ GeV but may make this model more difficult to detect at LHC than usual NS.

To illustrate how low EWFT comes about even with rather heavy top squarks, we show in Fig. 2 the various third generation contributions to Σ^u_u, where the lighter mass eigenstates are shown as solid curves, while heavier eigenstates are dashed. The sum of all contributions to Σ^u_u is shown by the black curve marked total. From the figure we see that for $A_0 \sim 0$, indeed both top squark contributions to Σ^u_u are large and negative, leading to a large value of $\Sigma^u_u(total)$, which will require large fine-tuning in Eq. (8). As A_0 gets large negative, both top squark contributions to Σ^u_u are suppressed, and $\Sigma^u_u(\tilde{t}_1)$ even changes sign, leading to cancellations amongst the various Σ^u_u contributions.
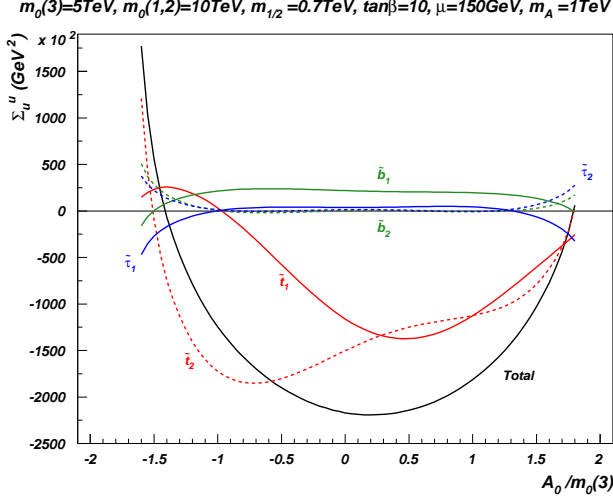
FIG. 2: Plot of third generation contributions to Σ_u^u versus A_0 for benchmark point RNS1 where solid curves come form the lighter mass eigenstate and dashed curves from the heavier. The black solid curve is Σ_u^u which has summed over all contributions.

V. RADIATIVE NATURAL SUSY AT COLLIDERS:

What chance does LHC have of detecting RNS? Unlike previous NS models, RNS has top and bottom squarks more typically in the $m_{\tilde{t}_1} \sim 1-2$ TeV and $m_{\tilde{t}_2} \sim 2-4$ TeV range, likely beyond LHC reach. It also has light higgsinos $\widetilde{W}_1^{\pm}$, $\widetilde{Z}_{1,2}$. While these latter particles can have substantial production cross sections at LHC, the mass gaps $m_{\widetilde{W}_1} - m_{\widetilde{Z}_1}$ and $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1}$ are typically in the $10-50$ GeV range. Thus, the visible decay products from $\widetilde{W}_1^+\widetilde{W}_1^-$ and $\widetilde{W}_1^{\pm}\widetilde{Z}_2$ production tend to be at rather low energies, making observability difficult. The best bet may be if gluinos lie in the lower half of their expected range $m_{\tilde{g}} \sim 1-5$ TeV. In this case, $\tilde{g} \to tb\widetilde{W}_1$ or $t\bar{t}\widetilde{Z}_i$ decays occur, and one might expect an observable rate for $\tilde{g}\tilde{g} \to 4t+E_T^{miss}$ signals. A portion of these events will contain cascade decays to $\widetilde{Z}_2 \to \widetilde{Z}_1\ell^+\ell^-$ and if the OS/SF dilepton pair can be reconstructed, then its distinctive invariant mass distribution bounded by $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1}$ may point to the presence of light higgsinos.

The hallmark of RNS and other NS models is the presence of light higgsinos with mass $m_{\widetilde{W}_1}$, $m_{\widetilde{Z}_{1,2}} \stackrel{<}{\sim} 200-300$ GeV. In this case, a linear e^+e^- collider operating with $\sqrt{s} \stackrel{>}{\sim} 2m_{\widetilde{W}_1}$ would be a *higgsino factory* in addition to a Higgs factory[12, 34]! The soft decay products from $\widetilde{W}_1 \to \widetilde{Z}_1 f\bar{f}'$ decay are not problematic for detection at an ILC, and will even be boosted as $\sqrt{s}$ increases well beyond threshold energy for creating charginos pairs. The reaction $e^+e^- \to \widetilde{Z}_1\widetilde{Z}_2$ will also be distinctive.

VI. MIXED AXION-HIGGSINO COLD DARK MATTER:

In R-parity conserving SUSY models with higgsino-like WIMPs[12], the relic density is usually a factor $10-15$ below the WMAP measured value of $\Omega_{CDM}h^2 \simeq 0.11$. This is due to a high rate of higgsino annihilation into WW and ZZ final states in the early universe. Thus, the usual picture of thermally produced WIMP-only dark matter is inadequate for the case of models with higgsino-like WIMPs.

A variety of non-standard cosmological models have been proposed which can ameliorate this situation. For instance, at least one relatively light modulus field is expected from string theory[35], and if the scalar field decays after neutralino freeze-out with a substantial branching fraction into SUSY particles then it will typically augment the neutralino abundance[36].

Alternatively, or in addition, if the strong CP problem is solved by the Peccei-Quinn mechanism in a SUSY context, then we expect the presence of axions in addition to R-parity odd spin $\frac{1}{2}$ axinos $\tilde{a}$ and R-parity even spin-0 saxions s. In this case, dark matter could consist of two particles: an axion-higgsino admixture[37–39]. The neutralinos are produced thermally as usual, but are also produced via thermal production followed by cascade decays of axinos at high T_R. The late decay of axinos into higgsinos can cause a re-annihilation of neutralinos at temperatures below freeze-out, substantially augmenting the relic abundance. In addition, saxions can be produced both thermally at lower range of PQ breaking parameter f_a, and via coherent oscillations at high f_a, and in fact may temporarily dominate the energy density of the universe. Their decay $s \to aa$ is expected to dominate and would add to the measured $N_{eff}(\nu)$. Their decay $s \to \tilde{g}\tilde{g}$ or $\tilde{a}\tilde{a}$ would augment the neutralino abundance, while late decays into SM particles such as gg would dilute all relics present at the time of decay. Exact dark matter abundances depend on the specific SUSY axion model and choices of PQMSSM parameters. It is possible one could have either axion or higgsino dominance of the relic abundance, or even a comparable mixture. In the latter case, it may be possible to directly detect both an axion and a higgsino-like WIMP.

VII. CONCLUSIONS:

Models of Natural SUSY are attractive in that they enjoy low levels of EWFT, which arise from a low value of μ and possibly a sub-TeV spectrum of top squarks and $\tilde{b}_1$. In the context of the MSSM, such light top squarks are difficult to reconcile with the LHC Higgs boson discovery which requires $m_h \sim 125$ GeV. By imposing naturalness using Δ_{EW} with weak scale parameter inputs, we allow for large cancellations in $m_{H_u}^2$ as it is driven to the weak scale. In this case, for some range of $m_{H_u}^2(m_{GUT}) > m_0$ (as in NUHM models), the weak scale value of $m_{H_u}^2$ will

be $\sim m_Z^2$ thus generating the natural SUSY model radiatively. Models with a large negative trilinear soft-breaking parameter A_t can maximize the value of m_h into the 125 GeV range without recourse to adding exotic matter into the theory. The large value of A_t also suppresses 1-loop top squark contributions to the scalar potential minimization condition leading to models with low EWFT and a light Higgs scalar consistent with LHC measurements. (More details on the allowable parameter space of RNS will be presented in Ref. [26].) The large negative A_t parameter can arise from large negative A_0 at the GUT scale.

While RNS may be difficult to detect at LHC unless gluinos, third generation squarks or the heavier electroweak-inos are fortuitously light, a linear e^+e^- collider with $\sqrt{s} \gtrsim 2|\mu|$ would have enough energy to produce the hallmark light higgsinos which are expected in this class of models. Since the model predicts a lower abundance of higgsino-like WIMP dark matter in the standard cosmology, there is room for mixed axion-higgsino cold dark matter. The axions are necessary anyway if one solves the strong CP problem via the PQ mechanism.

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